KINEMATIC, INTEGRAL, AND THERMAL CHARACTERISTICS OF TURBULENT STREAM OF GASEOUS SUSPENSION UNDER NONSTEADY THERMAL CONDITIONS

A. V. Fafurin and K. R. Shangareev

UDC 536.24:533.6.011

The effect of two-phasality and of thermal nonsteadiness on the characteristics of a stream of gaseous suspension is analyzed on the basis of the solution to integral equations of energy and momentum.

We consider nonsteady flow of a two-phase stream (gas + solid particles) in the initial segment and in the core segment of a pipe. The profiles of velocity and concentration at the channel entrance are assumed to be uniform and axisymmetric. At the entrance already there begin to form along the channel wall a thermal boundary layer and a dynamic one. With increasing distance from the entrance, the thicknesses of both boundary layers increase until they become equal to the pipe radius. The distance from the channel entrance where this occurs will be called the length of the initial segment or of the stabilization zone. The stream through the initial segment consists of a potential core and a boundary layer. The stagnation parameters P*, T* and the concentration of solid particles in the potential core remain constant and the same as at the channel entrance. The turbulence induced by the stream can change with acceleration or retardation of the gas, or as a result of any other external action. We will assume a turbulent flow throughout the entire channel space.

With no mass transfer between carrier phase and suspended phase taking place, the systems of equations for the gas and for the solid particles can be written separately [1-3]:

a) equation of continuity

$$\frac{\partial \rho_r}{\partial t} + \frac{\partial \rho w_x r}{\partial x} + \frac{\partial \rho w_r r}{\partial r} = 0; \tag{1}$$

b) equation of motion

$$\rho \frac{\partial w_x}{\partial t} + \rho w_x \frac{\partial w_x}{\partial x} + \rho w_r \frac{\partial w_x}{\partial r} = -\frac{dP}{dx} + \frac{1}{r}\frac{\partial r\tau}{\partial r} + F\rho_s\beta(w_{sx} - w_x); \tag{2}$$

c) equation of energy

$$\rho \frac{\partial}{\partial t}\left(h^* - \frac{P}{\rho}\right) + \rho w_x \frac{\partial h^*}{\partial x} + \rho w_r \frac{\partial h^*}{\partial r} = \frac{1}{r}\frac{\partial rq}{\partial r} + \Phi\rho_s\left(\frac{c_s}{c_p}h^* - h_s\right); \tag{3}$$

for the gas and

a) equation of continuity

$$\frac{\partial \rho_s w_{sx} r}{\partial x} + \frac{\partial \rho_s w_{sr} r}{\partial r} = 0; \tag{4}$$

b) equation of motion

$$\rho_s \frac{\partial w_{sx}}{\partial t} + \rho_s w_{sx} \frac{\partial w_{sx}}{\partial x} + \rho_s w_{sr} \frac{\partial w_{sx}}{\partial r} = F\rho_s(w_x - w_{sx}); \tag{5}$$

c) equation of energy

S. M. Kirov Kazan Chemicotechnological Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 43, No. 5, pp. 748-753, November, 1982. Original article submitted August 31, 1981.

 0022-0841/82/4305-1220$07.50

$$\rho_s \frac{\partial h_s}{\partial t} + \rho_s w_{sx} \frac{\partial h_s}{\partial x} + \rho_s w_{sr} \frac{\partial h_s}{\partial r} = \Phi \rho_s \left(\frac{c_s}{c_p} h^* - h_s \right) \tag{6}$$

for the particles.

The integral method of solution and analysis, with appropriate transformation of Eqs. (1)-(3), yields the transient-state energy and momentum relations for the flow of a nonisothermal and nonsteady, in the hydromechanical sense ($\partial w_0/\partial t \neq 0$) as well as in the thermal sense ($\partial h_0^* / \partial t \neq 0$; $\partial h_w/\partial t \neq 0$) two-phase gas stream through a pipe [4]. In the absence of such perturbing factors, these relations become the well-known steady-state ones derived elsewhere [5, 6].

We will consider the situation where the flow rate of gas through the channel remains constant and transiency is produced solely by variation of the temperatures of the stream core and of the streamlined surface: G = const, T_W = var, and T_0^* = var. Despite the constant flow rate, the velocity at the channel entrance will vary in time because of variation of the density. indeed, G = const implies also a constant mass velocity $\rho_{01} w_{01}$. As the temperature of the stream core T_0^* varies in time, so does the density ρ_{01} and thus also the velocity. With $\partial T_0^*/\partial t$ being the rate of change of temperature at the entrance, the time derivative of velocity is

$$\frac{dw_{01}}{dt} = \frac{w_{01}}{T_0^*} \frac{dT_0^*}{dt}. \tag{7}$$

Inserting expression (7) into Eq. (4) and the integral relations for momentum and energy, we obtain the final equation of motion

$$dW_0/dX = \left[4HW_0^2 \frac{C_f}{2} - H(W_0 - 1) \frac{2r_0}{w_{01}} \frac{1}{T_0^*} \frac{dT_0^*}{dt} - \right.$$

$$\left. - 4H \frac{\rho_s}{\rho_{01}} \frac{2r_0}{w_{01}} FW_0^2 \frac{\delta^{*\prime}}{2r_0} \beta \left(1 - \frac{w_s}{w_0} \frac{\delta_s^*}{\delta_s^{*\prime}}\right) \right] \Big/ \left[W_0 - \frac{W_0(W_0 - 1)}{H} \frac{dH}{dW_0} + (1 + H)(W_0 - 1) \right];$$

$$\delta_s^* = \int_0^{r_0} \left(1 - \frac{\rho_s w_s}{\rho_0 w_0}\right) \left(1 - \frac{y}{r_0}\right) dr;$$

$$\delta_s^{*\prime} = \int_0^{r_0} \left(1 - \frac{\rho_s w_x}{\rho_0 w_0}\right) \left(1 - \frac{y}{r_0}\right) dr; \tag{8}$$

equation of continuity

$$\frac{d}{dX} \left(4H \frac{N_{Re^{**}}}{N_{Re}}\right) W_0 = \left(1 - 4H \frac{N_{Re^{**}}}{N_{Re}}\right) \frac{dW_0}{dX} - \frac{2r_0}{w_{01}} \left(\frac{1}{T_0^*} \frac{dT_0^*}{dt} - \right.$$

$$\left. - 4H \frac{N_{Re^{**}}}{N_{Re}} \frac{1}{\psi_h^2} \frac{d\psi_h}{dt} - 4H \frac{N_{Re^{**}}}{N_{Re}} \frac{1 - \psi_h}{\psi_h} \frac{1}{T_0^*} \frac{dT_0^*}{dt} \right); \tag{9}$$

and equation of energy

$$\frac{dN_{Re_h^{**}}}{dX} = N_{St} N_{Re_1} W_0 + \frac{2r_0}{w_{01}} \frac{N_{Re^{**}}}{(T_0^* - T_w) W_0} \left[\frac{N_{Re_1} W_0}{4N_{Re^{**}}} + \right.$$

$$+ H'(1 - \psi_h) \frac{dT_0^*}{dt} - \frac{N_{Re_h^{**}}}{(T_0^* - T_w)} \frac{H'}{W_0} \frac{2r_0}{w_{01}} \frac{dT_0^*}{dt} + \frac{H'}{W_0} \frac{2r_0}{w_{01}} \times$$

$$\times \frac{N_{Re_h^{**}}}{(T_0^* - T_w)} \frac{dT_w}{dt} + \frac{N_{Re_h^{**}} H'}{W_0} \frac{2r_0}{w_{01}} \frac{1}{T_0^*} \frac{dT_0^*}{dt} + \frac{2r_0}{w_{01}} \frac{N_{Re_1} W_0^2}{8} \times$$

$$\times \frac{w_{01}^2}{g(h_0^* - h_w)} \frac{1}{T_0^*} \frac{dT_0^*}{dt} + \frac{3}{2} \beta \frac{N_{Nu_s} D^2 (1 - T_s/T_0^*)}{N_{Pr} d_s^2 (1 - \psi_h)}; \quad H' = \delta^{*\prime}/\delta_h^{**}. \tag{10}$$

The system of Eqs. (8)-(10) will be closed with expressions for the form factor $H = \delta^*/\delta^{**}$, the friction coefficient, the heat transfer coefficient, the relative velocity $w_0 - w_S$, and the relative temperature $T_0 - T_{S(t)}$ [4]

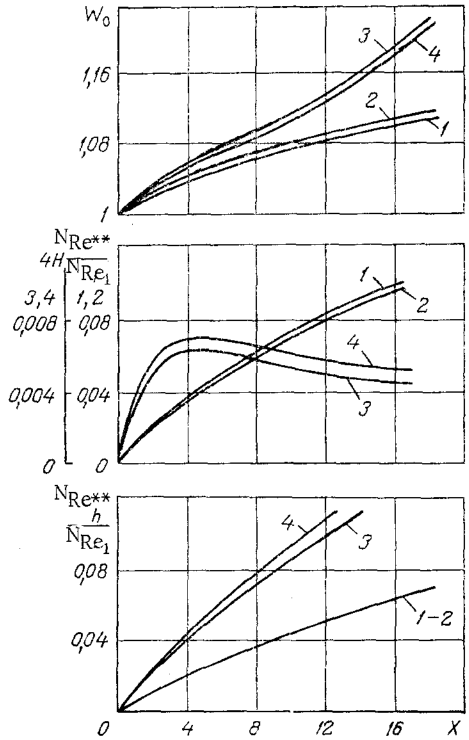

Fig. 1. Variation of kinematic, integral, and thermal characteristics along initial segment ($N_{Re,1}$ = 75,000, ψ_h = 0.47): 1) steady flow; 2, 3, 4) dT_w/dt = 230 K/sec; 3, 4) $dT_0^*/dt = 1.35 \cdot 10^4$ K/sec; 4) $\beta = 10^{-4}$.

$$\sqrt{\frac{C_f}{2}} = \int_{\omega_1}^{1} \sqrt{\bar{\rho}}\, d\omega \Big/ \int_{\xi_1}^{1} \sqrt{\bar{\tau}/\bar{\tau}_0}\, \frac{d\xi}{\varkappa\xi}; \tag{11}$$

$$N_{St} = \sqrt{\frac{C_f}{2}} \int_{\vartheta_1}^{1} \sqrt{\bar{\rho}}\, d\vartheta \Big/ \int_{\xi_{1h}}^{1} \frac{\bar{q}}{\bar{q}_0} \sqrt{\frac{\bar{\tau}_0}{\bar{\tau}}}\, \frac{d\xi_h}{\varkappa\xi_h}; \tag{12}$$

$$1 - \frac{w_s}{w_0} = \exp\left(-\frac{18\mu t g}{d_s^2 \rho_s}\right); \tag{13}$$

$$\vartheta = 1 - \frac{T_{s(t)}}{T_0} = \left(1 - \frac{T_{s(0)}}{T_0}\right) \exp\left(-\frac{6 N_{Nu_s} \lambda_0 t}{d_s^2 \rho_s c_s}\right). \tag{14}$$

The graph in Fig. 1 depicts the results of calculations, in four variants, based on Eqs. (8)-(10). The input parameters in the first variant are X = 0, $N_{Re,1}$ = 75,000, W_0 = 1, $4HN^{**}_{Re}/N_{Re}$ = 0.0007, ψ_h = 0.47, dT_0^*/dt = 0, and dT_w/dt = 0, corresponding to conventional steady nonisothermal flow in the initial pipe segment. A comparison of these results with those of other studies [7, 8] reveals a satisfactory agreement. Such a comparison had to be made, because in those other studies the energy equation was integrated assuming equal fourth roots of the Reynolds numbers based on energy thickness and momentum thickness respectively. The system of equations in this study was integrated by the Runge–Kutta method for given initial conditions.

The input parameters in the second variant were the same, except the time derivative of the wall temperature dT_w/dt = 230 K/sec.

This variant thus represents the situation of an instantaneous change in heat load or in flow pattern with T_0^* = const and T_w = var in time.

The results of calculations for this variant are depicted in Fig. 1 by curves 2. Evidently heating the surface under transient conditions causes the dimensionless group $4HN^{**}_{Re}/N_{Re}$ to decrease below its steady-state value. This is attributable to the way in which the boundary layer responds to thermal transiency. Heating the streamlined surface causes the density profile to become more uniform and the dimensionless group $4HN^{**}_{Re}/N_{Re}$ to increase, but the thickness of the boundary layer will decrease more rapidly so that the displacement thickness $2\delta^*/r_0$ also decreases. Despite that, however, the velocity in the potential stream is somewhat higher and this is related to an increase of the friction coefficient, inasmuch as to a smaller N^{**}_{Re} corresponds a larger C_f (Eq. (8)).

The values of the Reynolds number $N_{Re,h}^{**}$ for the thermal boundary layer are almost the same at a given level of the logarithm of the derivative of the surface temperature with respect to time and given level of the enthalpy factor. This has been confirmed by the results of experimental studies on heat transfer [9].

In the third variant it has been assumed that the time derivative of the temperature in the stream core is not zero, with all other parameters the same as in the second variant. The emergence of this derivative is equivalent to acceleration of the stream at the channel entrance and to a transient velocity gradient as an added prior influencing factor. The analogy to a negative longitudinal pressure gradient suggests that transient acceleration should suppress the boundary layer and decrease the enthalpy factor and, as a consequence, also decrease the displacement thickness. This trend is, indeed, noted on the center graph in Fig. 1. The larger increase of velocity in the potential core (center graph) is solely due to change in density, because, according to the equation of continuity, a change in velocity is associated not only with the evolution of the displacement thickness $2\delta^*/r_0$ but also with the time derivative density:

$$W_0 = \frac{1 - \int\limits_0^x \frac{1}{\rho_{01} w_{01}} \frac{\partial}{\partial t} \rho_0 \left[1 + 2 \frac{\delta^{*\prime}}{r_0} (1 - \psi_h) \right] dx}{(1 - 2\delta^*/r_0)} . \tag{15}$$

where $\delta^{*\prime} = \int\limits_0^{r_0} \frac{\rho}{\rho_0} \left(1 - \frac{w_x}{w_0}\right)\left(1 - \frac{y}{r_0}\right) dr.$

As the heat load increases ($dT_0^*/dt > 0$), the velocity will increase faster. When $dT_0^*/dt < 0$, then the relative velocity is lower than in the steady state.

When the velocity profile of the dynamic boundary layer changes, then an increase of the Reynolds number in the thermal boundary layer is due to a more profound distortion of the density and temperature profile causing an additional increase of $N_{Re,h}^{**}$.

Finally, the fourth variant corresponds to a two-phase stream with solid particles underheated ($T_s/T_0 \sim 0.97$) and 10% slower. Their volume concentration in the stream has been assumed here to be $\beta = 10^{-4}$.

It has been established that the given parameters characterizing two-phasality do not contribute significantly to the increase or decrease by a few percent of kinematic, thermal, and integral characteristics relative to those in the third variant. The results of these calculations indicate, however, the trend of the effect of two-phasality.

NOTATION

c, specific heat; C_f, friction coefficient; D, channel diameter; d_s, diameter of solid particles; g, acceleration of gravity; h, enthalpy; R_{Nu}, Nusselt number; P, pressure; N_{Pr}, Prandtl number; q, thermal flux density; r, radius; N_{Re}, Reynolds number; N_{St}, Stanton number; t, time; T, temperature; w, velocity; $W_0 = w_0/w_{01}$, relative velocity; x, longitudinal coordinate; X = x/D, referred longitudinal coordinate; y, transverse coordinate; β, volume concentration of solid particles; δ*, displacement thickness; δ**, momentum thickness; δ_h^{**}, energy thickness; ϑ, dimensionless temperature; λ, thermal conductivity; μ, dynamic viscosity; ρ, density; τ, shearing stress; ψ_h, enthalpy factor; superscript * refers to stagnation; subscripts h refers to the thermal boundary layer, w refers to the wall (surface), 0 refers to the region outside the boundary layer; 01 refers to channel entrance, x refers to the longitudinal coordinate, r refers to the radial coordinate, and s refers to the solid phase.

LITERATURE CITED

1. A. A. Glazunov et al., "Interaction of boundary layer at burning surface and nonequilibrium two-phase stream in axisymmetric deLaval nozzle," Zh. Prikl. Mekh. Tekh. Fiz., No. 3, 53-62 (1977).
2. A. K. Lyunin, Blizzard Mechanics [in Russian], Siberian Branch, Academy of Sciences of the USSR, Novosibirsk (1963).
3. L. E. Sternin, Basic Gas Dynamics of Two-Phase Streams in Nozzles [in Russian], Mashinostroenie, Moscow (1974).
4. A. V. Fafurin, K. R. Shangareev, and R. A. Muslimov, "Transient heat transfer in two-phase stream along initial pipe segment," in: Heat and Mass Transfer in Chemical Technology [in Russian], Izd. Kazan. Khim.-Tekhnol. Inst., Kazan (1979).
5. A. I. Leont'ev, "Limiting-value laws of friction and heat transfer in turbulent boundary layer," Doctoral Dissertation, Technical Sciences, Novosibirsk (1962).
6. H. Schlichting, Theory of Boundary Layer, McGraw-Hill (1968).
7. A. I. Leont'ev, B. P. Mironov, and A. V. Fafurin, "Turbulent boundary layer of dissociated gas in initial pipe segment," Zh. Prikl. Mekh. Tekh. Fiz., No. 100-105 (1967).
8. S. S. Kutateladze (ed.), Heat and Mass Transfer and Friction in Turbulent Boundary Layer [in Russian], Siberian Branch, Academy of Sciences of the Novosibirsk (1964).
9. A. V. Fafurin and K. R. Shangareev, "Transient heat transfer in axisymmetric channel," in: Heat and Mass Transfer in Aircraft Engines [in Russian], No. 178, Izd. Kazan. Aviats. Inst., Kazan (1974), pp. 7-12.

UNIVERSAL EQUATION OF TRANSIENT PLANE JET IN CONCURRENT STREAM

O. N. Bushmarin, E. Yu. Egorova, and N. F. Trubitsyn

UDC 532.526.2

An equation is derived and subsequently integrated which is "universal" not only with respect to velocity of the concurrent stream and initial conditions of jet discharge but also with respect to choice of characteristic scale for the transverse coordinate.

A universal equation for a transient laminar jet and a transient gradiental concurrent stream of incompressible fluid has been derived in an earlier study [1] without the use of any integral relations, i.e., in purely differential form. We will now write this equation and the boundary conditions for the dimensionless flow function φ in the form

$$B^2\frac{\partial^3\varphi}{\partial\eta^3}-(r_{01}+f_{00}r_{10}+f_{10})\frac{\partial\varphi}{\partial\eta}+\left(\frac{g_{01}+f_{00}g_{10}}{2}+f_{10}\right)\eta\frac{\partial^2\varphi}{\partial\eta^2}-$$

$$-r_{10}\left(\frac{\partial\varphi}{\partial\eta}\right)^2+\left(\frac{g_{10}}{2}+r_{10}\right)\varphi\frac{\partial^2\varphi}{\partial\eta^2}=\sum_{k=n=i=j=l=m=0}^{\infty}\left[-\frac{\partial^2\varphi}{\partial f_{kn}\partial\eta}K+\right.$$

$$+\frac{\partial^2\varphi}{\partial r_{ij}\partial\eta}L+\frac{\partial^2\varphi}{\partial g_{lm}\partial\eta}M+\left(f_{00}+\frac{\partial\varphi}{\partial\eta}\right)\left(\frac{\partial^2\varphi}{\partial f_{kn}\partial\eta}N+\frac{\partial^2\varphi}{\partial r_{ij}\partial\eta}P+\right.$$

$$\left.\left.+\frac{\partial^2\varphi}{\partial g_{lm}\partial\eta}Q\right)-\frac{\partial^2\varphi}{\partial\eta^2}\left(\frac{\partial\varphi}{\partial f_{kn}}N+\frac{\partial\varphi}{\partial r_{ij}}P+\frac{\partial\varphi}{\partial g_{lm}}Q\right)\right],\qquad(1)$$

$$\varphi=\frac{\partial^2\varphi}{\partial\eta^2}=0\ \text{for}\quad\eta=0;\quad\frac{\partial\varphi}{\partial\eta}=0\ \text{for}\quad\eta\to\infty;$$

M. I. Kalinin Leningrad Polytechnic Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 43, No. 5, pp. 753-759, November, 1982. Original article submitted August 6, 1981.

0022-0841/82/4305-1224$07.50